Analyzing the Optimal Density and Pollution Level of a Monocentric City

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Abstract

In this paper we build a urban-rural model to discuss the location of working places in a monocentric city. The pollution and congestion externality are added into the model. Government collects environment protection tax to accomplish anti-pollution actions to raise the utility level of representative workers. We use four major cities in Asia to do numerical simulation to provide us some specific evidences. It can be concluded that population density is increased and city size becomes smaller after externality is considered. The aggregate rent is reduced since part of the disposable income is used to dealing with negative externality of pollution.

Keywords: Pollution; Population Density;

Contents

1 Introduction

A higher density of a city is sometimes considered to contribute a negative externality since workers tend to live in the less congested area. Taking Hong Kong as an example, a person has to endure high density and low living area but are still willing to pay tens of millions of dollars to purchase a two-bedroom apartment in the narrow city downtown. In this example, the high population density seems to exert negative impact to the utility as the employees have to sacrifice more in living condition. But from the aspect of urban land market, a higher level of density, particular near the center of a city, reduces the commuting cost and consequently produces positive externatity to workers. Meanwhile empirical results have shown that a bigger city with more population usually displays higher production efficiency(Y Kanemoto, T Ohkawara(1996)). These topics remain us huge space to cover research on the optimal city size and density level.

Pollution is another critical issue in urban development. Taking the pollution level of New Delhi and Beijing as examples, these two largest cities of the two largest developing countries around the world exhibit a high level of air pollution during the recent decades, which hence show negative externality to the working conditions. Some companies and government authorities such as Japanese Embassy pay high level of health compensation to their employees in these heavy-polluted cities so as to make these working places attractive. It remains a challenging task for scholars to take pollution level of a urban city into account when discussing the optimal urban size and density problem. Existing papers(Steemers K. 2003) demonstrates that a high density level will inevitably increase automobile usage and consequently make the cleaness level even worse. But recent empirical research has shown that Europe big cities fail to display significant high level of pollution comparing to small cities. The reason could be local authorities make strict anti-pollution policies and regulations after the heavy pollution in the 1960s. Empirical works (James Liang, 2008) figures out the pollution level keeps increasing before output level reachs 10000 USD per capita while after this threshold the pollution level starts to decrease. It could be explained that before accumulating certain amount of wealth, the local authorities have no funding fighting with pollution while after 10000 USD threshold under public concerns the authorities have the money to cure the contamination problem.

In this paper it should be discussed on the topic of urban density and city size problem and some externality such as congestion and pollution are taken into account. We should compare the result of city size and density when we endogenizing congestion and pollution. Since the analytical calculation is hard to reach, some numerical simulation will be done in the last part to provide us some detailed evidences of the effect of contamination and pollution influence.

2 Literature Review

There are quite amount of literatures discussing the optimal city size and denstiy problem. Some of the past literature figure out that there might exist a threshold point for productivity of a particular city. Y Kanemoto, T Ohkawara(1996) point out that the productivity increases by 7 per cent with population more than 400,000 while increases only 1 per cent in cities with population less than 200,000 when calculating the agglomeration effect of the economy in Japan. Also in this paper the author could not show Tokyo is too large by testing Henry George Theorem, which seems quite counterintuitive. WC Wheaton(1997) uses simulation method to show that the density in the center part of optimal cities should be greater than the level of magnitude in market cities, which supports Kanemoto's counterintuitive result. J Eaton, Z Eckstein(1997) also figure out that large cities usually have comparably higher level of human capital, higher wage rate and lease rate given the condition that employees are available to migrate between cities by studying the cases in France and Japan. R Arnott(2004), on the other hand, suggests that applying Henry George Theorem into the study of the optimal city size is still deserved for further research and Kanemoto's research on Tokyo should be accorded considerable weight in policy circles. However, R Capello, R Camagni(2000) argues that there could be a misleading trend in the topic of optimal city size. He defines the concept of "efficient size" instead of "optimal size" and indicates that economy of scale continues to exist until a certain level by doing empirical works in 58 Italian cities. CC Au, JV Henderson(2006), suggests a large fraction of cities in China are undersized because of the government control policy. Most people could be better off under the inverted-U shape real income per worker curve with respect to the city size(Henderson 1974, Helsley and Strange 1990 , Krugman, Venables 1999, Duranton and Puga 2001) when estimating the net urban agglomeration economies for cities.

The abovementioned results quite support the reality that an increasing number of people live in big and more densified cities. However, there are multiple problems such as pollution and congestion in the city which might be solved by government policy and interaction. JV Henderson(1974) suggests that taxing an externality will make people in a city better off, which will consequently induce increasing number of workers from other cities and make the city even bigger. Y Song, Y Zenou(2006), on the other hand, suggests that a higher property tax actually leads the city size to become smaller through reducing the population density. They achieve an empirical result that when the property tax increases by 1%, the city size would decrease by 0.4%. A Anas, R Xu(1999) states that centralizing effect of toll on workers outweighs decentralizing effect of toll on employers. As a result, a dispersed city will have to endure a higher level of population density. The effect of congestion and pollution could be complex when analyzing their effect towards the optimal city size when considering government involvement.

3 Model Setup

In this section we consider a general closed city model. The city is divided into two components: the urban part and the rural part. Without loss of generality we may assume the city is in circle size and the total area of urban and rural is fixed, which is marked by A. Citizens freely choose to live in the urban or rural area by comparing the utility. When the utility level living in the urban outweighs his/her retained utility, he/she rationally chooses to live in the urban. Otherwise he/she will choose to live in the rural. The retained utility level U_0 is endogenously decided by the exogenous aggregate population of urban(N). The border of the city(the distance of border from CBD) is endogenously calculated in the optimal decision problem.

We may also assume that citizens could move freely among the urban area and there does not exist any moving cost. As a result, workers could choose wherever to work to maximize the utility level. All workers have to work at the center of urban area (Central Business District) and commute every day. A represetative employee has to decide how much to spend between commute cost T, other consumption z which is numeraire good and land consumption. In our model when we endogenize the level of city pollution, the utility of a worker also depends on the cleaness index level (p). That is to say, a citizen will choose to maximize

$$U(q, z, p) \tag{1}$$

subject to

$$R(t)q + z + T(t) = y(1 - g) \tag{2}$$

Where q is the land amount, z is the amount of other consumptions and p is the cleaness quality index. $T(t)$ represents commute cost in which is obviously strictly increasing with repect to the distance from CBD t. $R(t)$ is the land rent rate and y is the uniform household income. g is the government pollution tax ratio. That is to say, the government collect gy to manage the overall pollution level. We may assume the pollution level is porportional to the money spent on the environment protection per area per capita, i.e.

$$p'(t) = \frac{C_1 gy}{An'(t)} \tag{3}$$

where C_1 is an exogenous city anti-pollution efficiency. A is the total area of the city and $n'(t)$ denotes the population density at distance t. We may add a border condition that the air cleaness level in the center is also determined by the above factors.

$$p(0) = \frac{C_0 gy}{An(0)} \tag{4}$$

Where n(0) denotes total population in the urban area. C_0 is another exogenous coefficient which determines pollution level of the CBD area of a city. At the same time it is assumed that the utility level displays a contant return to scale characteristics.

The owner of the land wants to maximize the aggregate rent. The landlord rents the land to the consumer who bids the highest price while the consumer of the land bids the price so that his retained utility does not exceed the utility he achieves under that price level. The opimal decision problem becomes

$$max_{z,q,b,g} \int_0^b [\frac{y(1-g) - T(t) - z(t)}{q(t)}] 2\pi t(1-v)dt + [A - vb^2]R + \beta_1[U(z, q, p) - U_0] \tag{5}$$

subject to

$$n'(t) = \frac{-2\pi t(1-v)}{q(t)} \tag{6}$$

$$T'(t) = \frac{C_2 n(t)}{2\pi t v} \tag{7}$$

$$p'(t) = \frac{C_1 g y}{A n'(t)} \tag{8}$$

Where we shall denote three costate variables $n(t)$, $T(t)$ and $p(t)$ as λ_1, λ_2 and λ_3.

n(t): total number of urban population beyond the distance t

T(t): commute cost for workers living at distance t from CBD where T(0)=0

p(t): environment quality index at distance t from CBD

v: fraction of urban land area to road and transportation

q(t): land consumption amount at t

U_0: retained utility level that worker is willing to live in the urban area

g: government tax ratio collected to protect the environment

z(t): other numeraire consumptions

b: border of the urban (i.e. the size of urban area)

$U(z, q, p)$: utility of household which has a property of constant return to scale

A: size of urban and rural land area

R: opportunity cost of land

In the optimal control problem (5), the first part of the expression is the aggregate rent collect in the urban area while the second part is the revenue in the rural area. In this closed city model, U_0 must be carefully select to ensure that the overall population in the urban area is N.

$$n(0) = N = \int_0^b \frac{2\pi t(1-v)}{q(t)} dt \tag{9}$$

Notice equation (6) is the derivative form of above condition (9). In the constraint (7) it is assumed that marginal transportation cost is proportional to population but inversely proportional to road occupation area. The control variables are z, q,b and g while three shadow prices (Lagrangian Multipliers) are the costate variables.

Following the optimal control theorem, the Hamiltonian is as followed.

$$
\begin{aligned}
H =& [\frac{y(1-g)-T(t)-z(t)}{q(t)}]2\pi t(1-v)+\beta_1[U(z,q,p)-U_0] \\
&+\lambda_1[\frac{-2\pi t(1-v)}{q(t)}]+\lambda_2[\frac{C_2 n(t)}{2\pi tv}]+\lambda_3[\frac{C_1 gy}{An'(t)}]
\end{aligned}
\tag{10}
$$

When we derive the first order condition with repect to environment tax ratio we have

$$
\frac{2\pi t(1-v)y}{q}=\lambda_3\frac{C_1 y}{An'(t)} \tag{11}
$$

The economic meaning of this equation is quite intuitive that the marginal cost of collecting tax(LHS) must be the same as the marginal revenue the agent gains when the enviroment becomes better under income environmental protection tax(RHS). Then we may focus on the three Envelop conditions:

$$
\dot{\lambda_1}=-\frac{\partial H}{\partial n}=-\lambda_2\frac{C_2}{2\pi tv}>0 \tag{12}
$$

$$
\dot{\lambda_2}=-\frac{\partial H}{\partial T}=\frac{2\pi t(1-v)}{q}>0 \tag{13}
$$

$$
\dot{\lambda_3}=-\frac{\partial H}{\partial p}=-\beta_1 U_p<0 \tag{14}
$$

It could be explained that $\lambda_3=\frac{2\pi t(1-v)An'(t)}{C_1 q}<0$ is the marginal social cost of land rent decreasing when we add one more additional portion of environment tax collection. The expresson clearly shows that the rent loss when taking enviroment protection into account is proportional to the total area because the environment index is calculated as environment protection fee per capita per area among the whole city(both urban and rural). C_1 could be regarded as a kind of protection efficiency level. It is certain both intuitively and from the expression of λ_3 that when such efficiency is increased, the social rent loss to protect the environment decreases. When the first order condition and envelop conditon are combined together, the relative price of land rent and environmental protection is displayed as follows:

$$
\frac{U_p}{U_z}=-\frac{A}{C_1}\frac{q(n'+tn'')-q'tn'}{q} \tag{15}
$$

$$
\frac{U_q}{U_z}=\frac{y(1-g)-T-z}{q}-\frac{\lambda_1}{q} \tag{16}
$$

From the above deduction it could be found that $\lambda_1 < 0$, $\lambda_2 < 0$, $\lambda_3 < 0$ and $\lambda_1' > 0$, $\lambda_2' > 0$, $\lambda_3' < 0$. At the border we must have the land rent collected equal to the land opportunity cost:

$$\frac{y(1-g) - T(b) - z(b)}{q(b)} = R \tag{17}$$

The relative utility of environmental protection with respect to numeraire good could be treated as the price of environmental protection. Notice that $\frac{U_p}{U_z} = -\frac{qA}{C_1}\frac{d}{dt}(\frac{tn'(t)}{q})$ which is exactly the marginal utility gained from collecting an extra dollar tax to environmental protection. At the same time, from (16) the relative land rent price is the marginal utility contribution of land. When the congestion problem does not exist, it could be deduced that $R(t) = \frac{y(1-g)-T(t)-z(t)}{q(t)}$. The difference term $-\frac{\lambda_1}{q} > 0$ is the endogenized social land cost under congestion.

In this problem, what the authority does is to collect environment tax gy to help handle pollution. However, the goal of the government is not setting a pollution standard. Actually, the government select a tax ratio g to maximize the landlord's aggregate house rent in the city. In this case, the optimal control problem might be transformed into a three good selection problem which the worker optimally chooses z,q and p under the constraint $R(t)q + z(t) + p_p p(t) = y$ where p_p is the corresponding price level of clean environment. In other cases, if the authority's goal is not to maximize land owner's utility but to keep the cleaness level above a certain standard, the above optimal control problem will crash and relative "price level" of environmental protection could fail to maintain.

When the shadow environment "price level" is compared between the CBD and the border of urban area. The conclusion is that the result is ambiguous since the cleaness index is proportional to the anti-pollution expense per area per capital. The density in CBD is relatively high so that each person who chooses to live near CBD does not pay so much attention on the environmental protection. If the protection efficiency only depends on the money spent per area, workers near CBD has more incentive to pay more tax to protect the environment.

The land rent when congestion and pollution are all endogenized could be compared when such we care nothing about pollution. Denote R_{nopo} and R_{withpo} as the land rent without and with contamination, it could be obtained that

$$\frac{R_{withpo}}{R_{nopo}} = 1 - \frac{gy}{y - T - z} < 1 \tag{18}$$

That is to say, the pollution is finally transferred to the loss of landowner's house rent. Beijing, recently, has provided examples supporting above statement. With the further deteriorating air quality and water pollution, the house rent has decreased since US embassy in Beijing declares "severe air pollution" in the capital city. Rich people tend to sell houses in Beijing at a lower price and then replace them with houses in North America cities to seek for higher level of cleaness. Such negative externality, particularly in developing countries, destroy both the interst of land owners and land rents by cutting down the aggregate land rent and disposable income. However, noticing that $\dot{\lambda_3} = -\beta_1 U_p$, the environment effect on land rent could be small if people do not care the environment condition. These could be explained into two different cases. One is that the overall pollution is very slight such as the situation in Canada. The other is the living standard is too low to concern too much on environmental protection such as the situation in Central Africa. In both these two cases $U_p \to 0$. But this two situations are quite different. Empirical studies(James Liang 2008) have shown that when the GDP per capita is near 10000 USD, people concern the environmental protection the most and the coresponding contamination level reachs the peak. Historical evidence in London, Los Angeles, Tokyo could demonstrate the variation curve of contamination.

Now we focus on the size of a closed city taking pollution into account.

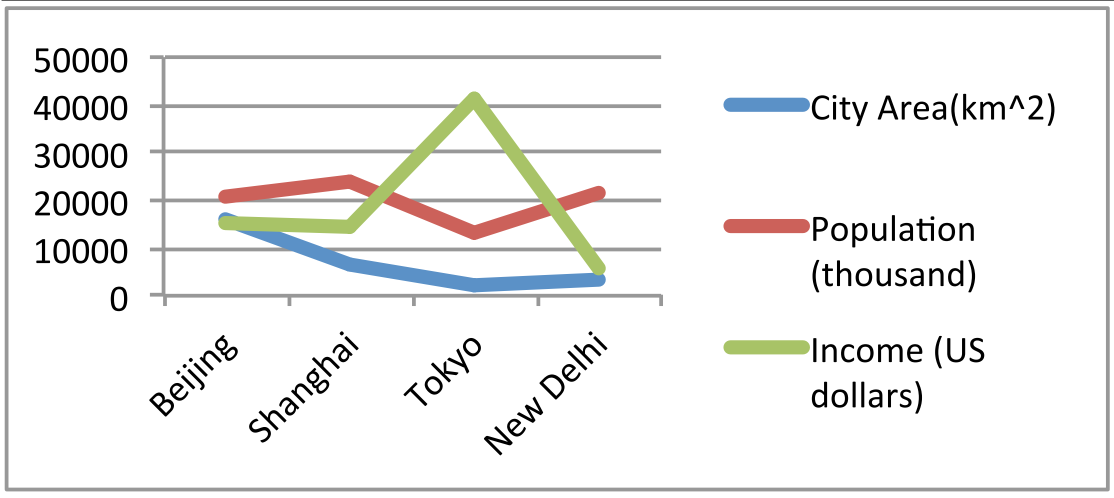
$$0 > \lambda_3 = \frac{2\pi t(1-v)An'(t)}{C_1 q} = -\frac{4\pi^2 t^2 (1-v)^2 A}{C_1 q^2} \tag{19}$$

When λ_3 declines from 0 (no pollution concerns) to negative (with pollution concerns), it is clear that the land consumption level q decreases. That is to say, the area near CBD has a greater density when people begin to care city pollution level. In order to maintain the population level N, the city should be smaller than we care nothing about pollution level. Empirical evidence around the world demonstrates that people are more willing to live near the center of cities in developing countries even willing to endure the poor environment quality. On the contrary, citizens in the developed countries such as United States are more willing to live in the suburb area with a clean atmosphere. Also, people living at the CBD are least affected by the environment protection policy since the cleaness level being enhanced is proportional to protection expence per area per capita. This could be an explanation why New Yorker concern less about New York's dirty environment since the density level in Manhattan is significantly high.

4 Numerical Simulation

In this section we will use some specific data to depict the impact on city density and size when considering congestion and pollution. Four Asian cities: Beijing, Shanghai, Tokyo and New Delhi are under numerical simulation discussion. Those four representative cities are on the different steps of environmental protection problem. For instance, Beijing and New Delhi both experience severe air and water pollution in recent years which the GDP per capita in Beijing is above 10000 USD while New Delhi is significantly lower than 10000 USD. On the contrary, Shanghai and Tokyo experience less air and water contamination but these two cities are still on the different income stage. For instance, Shanghai's GDP per capital is around 10000 USD while Tokyo is far above 10000. Japan has experienced severe pollution in the last century but the environment quality started to increase in the recent decades. The comparison of these four cities remain us an interesting topic figuring out the different effect of congestion and contamination under different income level.

City	Beijing	Shanghai	Tokyo	New Delhi
City area (km^2)	16410	6340	2187	3716
Population (thousand)	21150	24150	13185	21700
Income (US dollars)	15000	14500	41100	5610



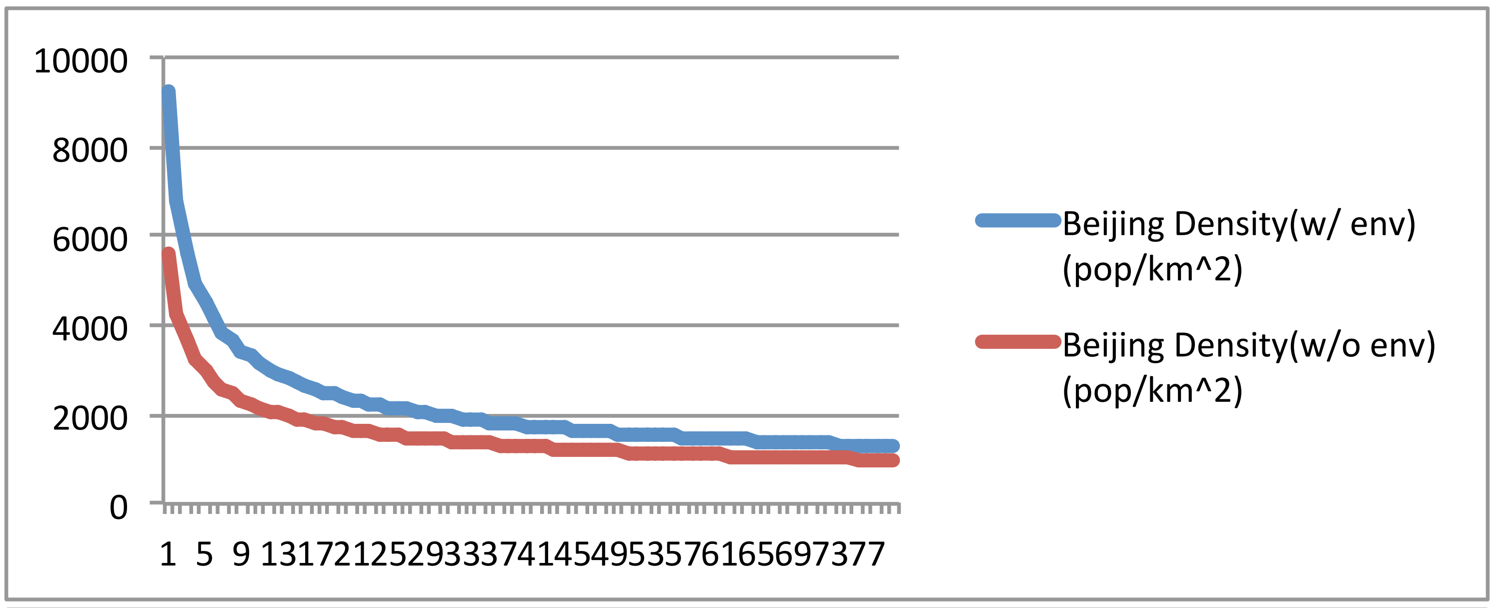
Then we may assume a specific form of utility function.

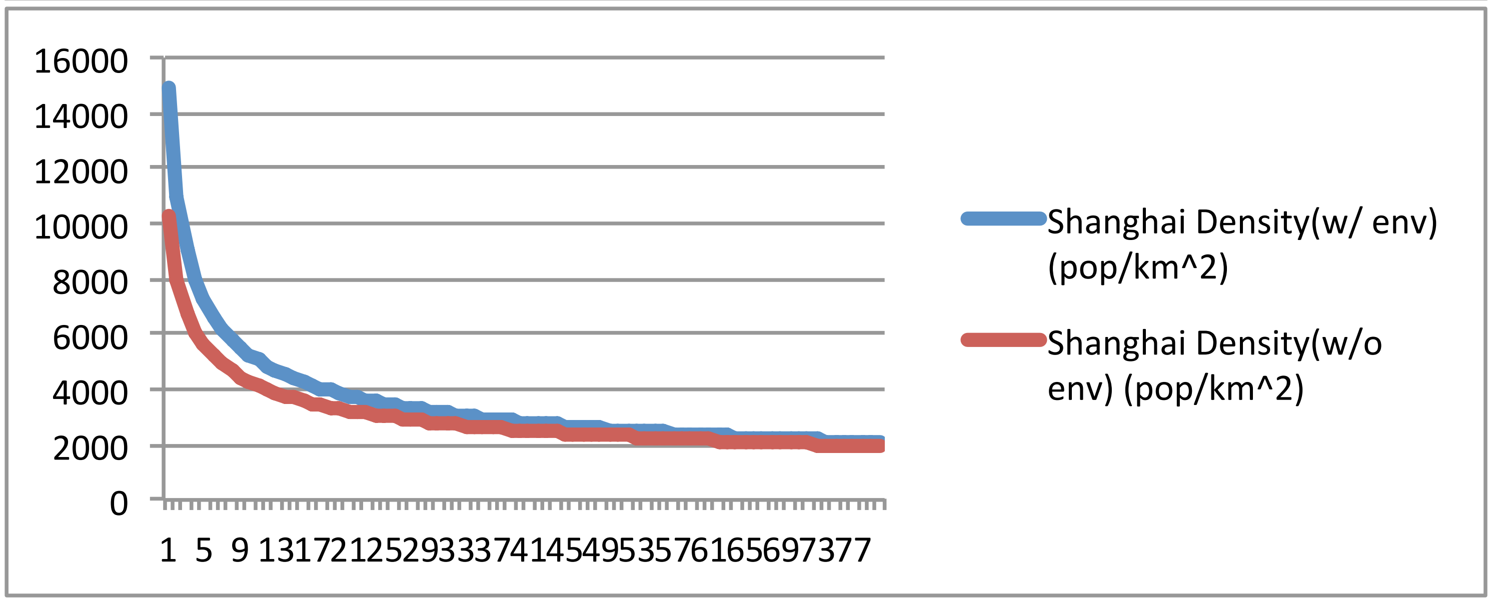
$$U(z, q, p) = z^{\alpha} q^{\beta} p^{\gamma} \tag{20}$$

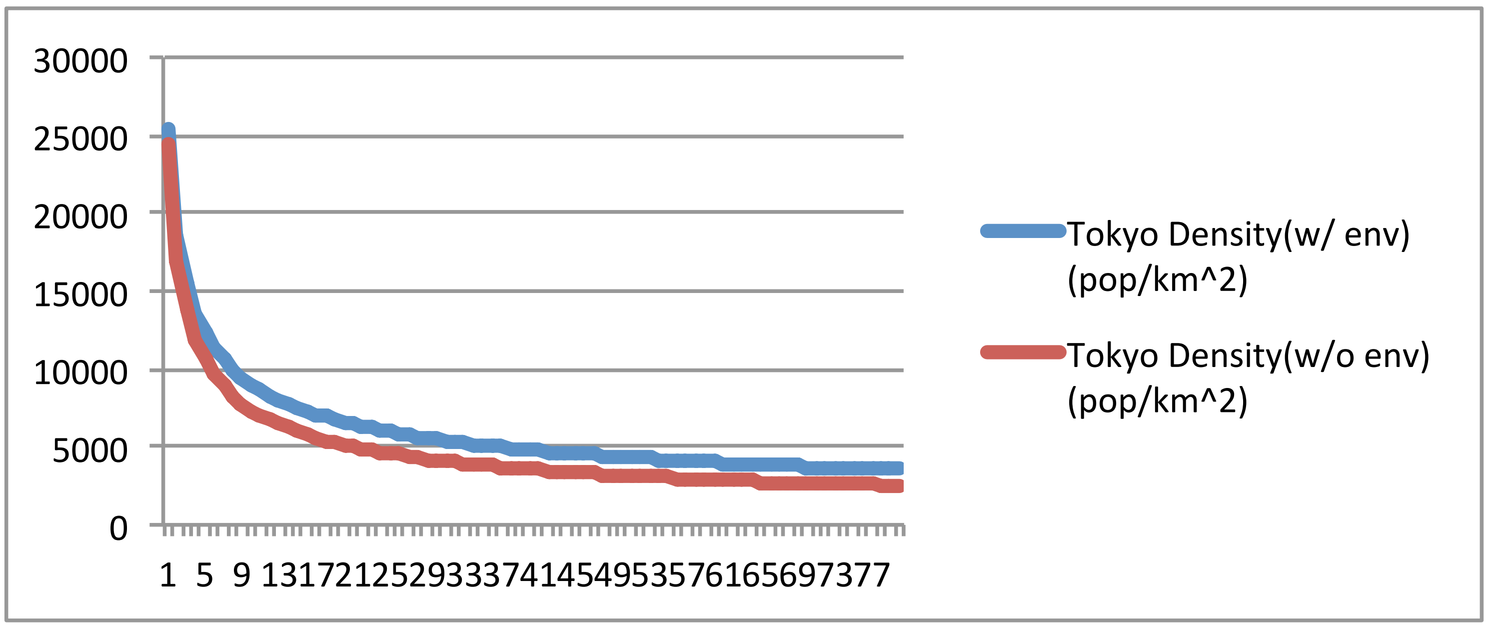
where $\alpha = 0.6$, $\beta = 0.3$ and $\gamma = 0.1$. Therefore, the utility function displays a contant return to scale property. These coefficient are supported by empirical evidence that citizens on average spend one-fourth to one-third of their income on house rent in eastern asia metropolitan cities. And the health and environment expenditure is approximately one-tenth of the aggregate income.

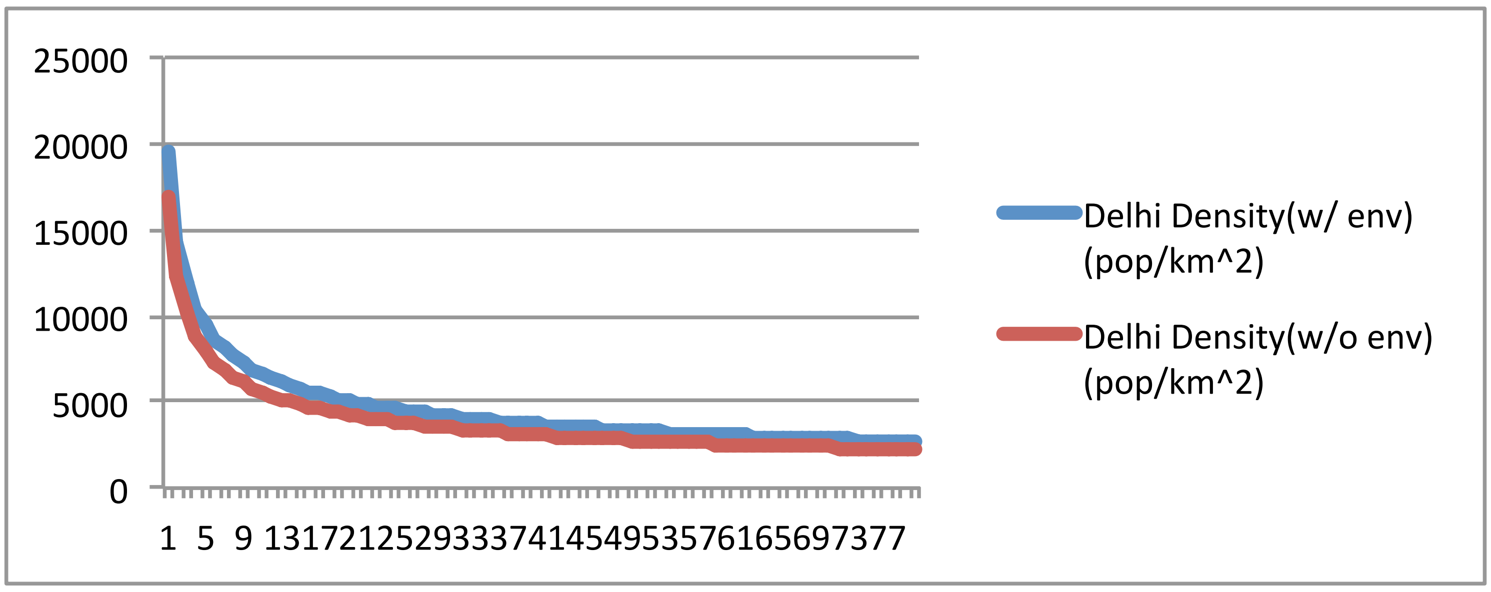
Another important factor here is to make sure the road occupation rate, protection efficiency coefficient and average transportation cost. Leduc G(2008) figures out that the average road occupation rate in East and South Asia is around 15 %, which is considerably lower than the rate in European and North American coutries. This could be explained by the expensive land price in Asia big cities. The citizens in Japan, China and India, however, tend to live in big cities since most of the fascinating job opportunities are in these metropolitans. The protection efficiency coefficient in this paper is decided by the Average Air Quality Index (AQI) during the recent period. Tokyo, under this measure, has the highest protection efficiency since its environment quality is the best around these four cities while Delhi is the worst because of the poorest Average Air Quality Index. When we estimating the commute cost coefficient in the numerical simulaton, it is a straight forward way to assume they are equal. Although it might not be the real case, but it provides us the most convenient way to finish numerical simulation process.

By combining the conditions (11) to (16), (19) and our proposed utility function and coefficients, we can reach the following results for these four cities.







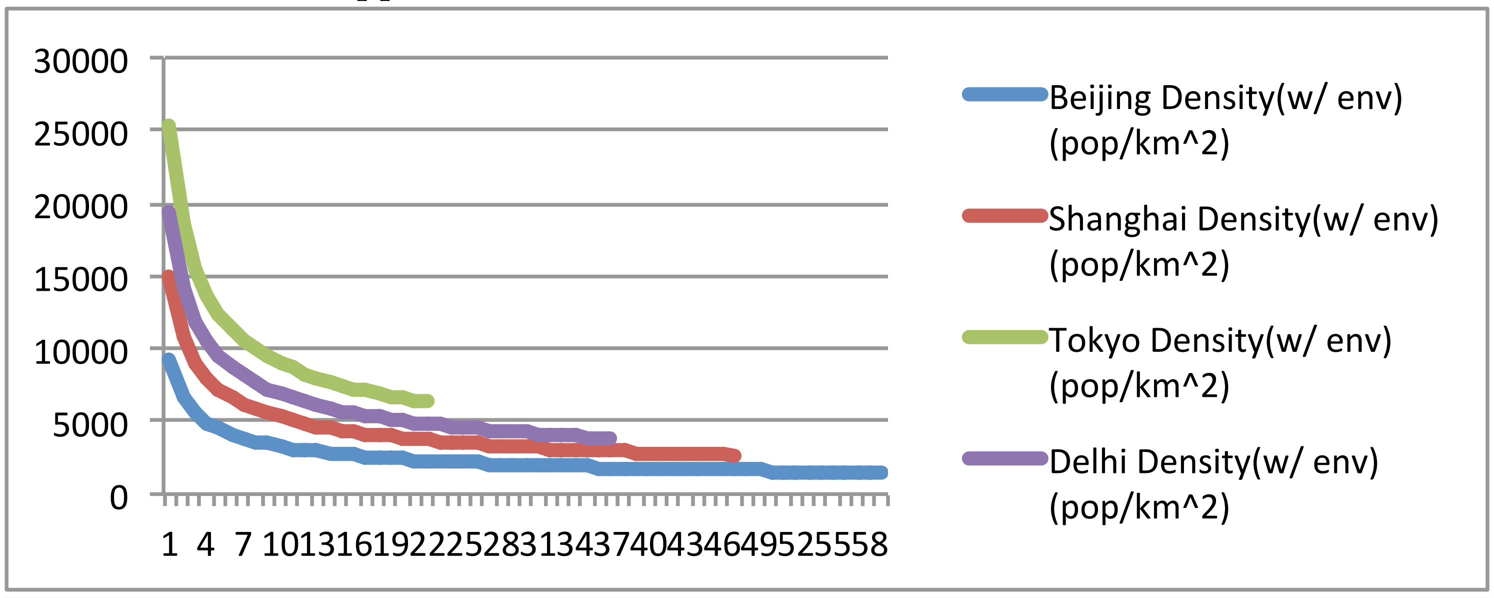


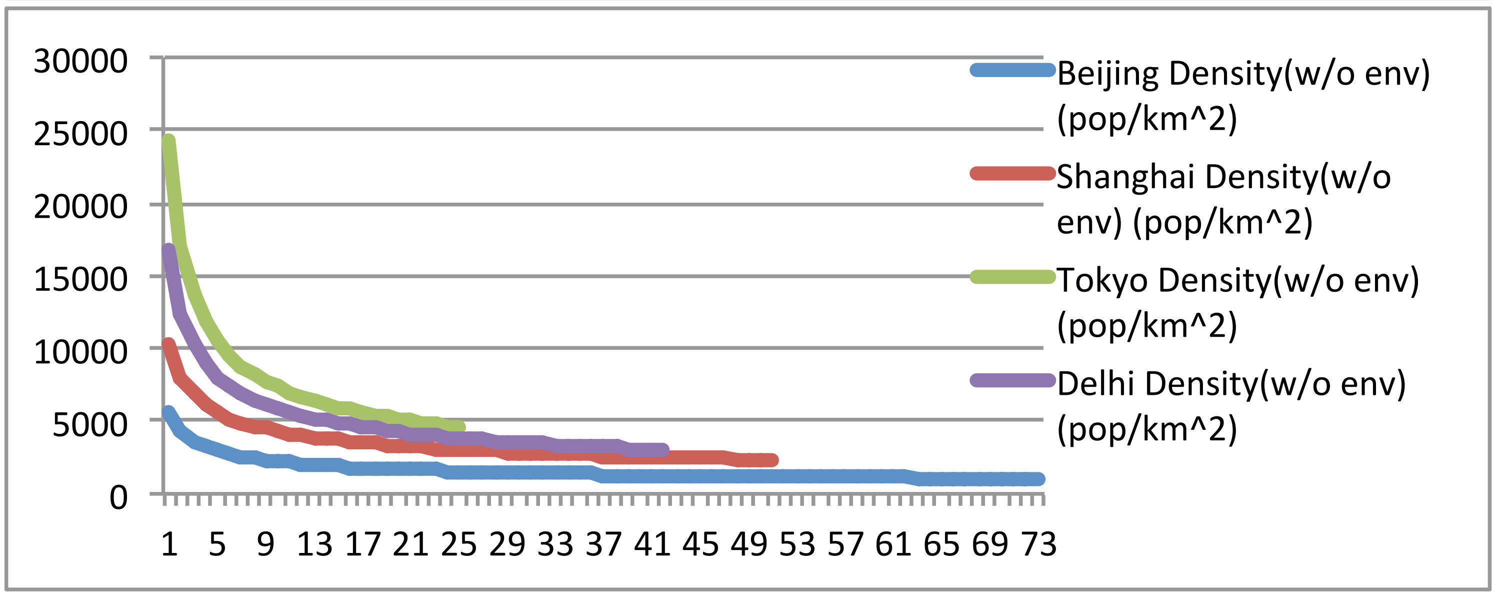
From above four graphs, it can be conluded that Beijing and Shanghai's population density are mostly affected by the environment protection tax, particularly near the center of the city. On the contrary, the population density near CBD in Tokyo and Delhi are not strongly affected by the anti-pollution actions. The area that is 5 to 15 kilometers away from CBD is mostly affected by the policy. People tend to stay away from these regions under anti-pollution policies since they might suffer the greatest impact as the protection efficiency is calculated as expense per area per capita.

In the following table we conclude the market and optimal size of these four cities. And then we compare the result to the actual size of cities using the real data and we may assume the size of cities are circles.

City	Beijing	Shanghai	Tokyo	New Delhi
Optimal Radius(w/ env)	59	47	22	37
Market Radius(w/o env)	73	51	25	42
Real Radius	72	45	26	34

From the above tables it can be concluded that the market size is bigger than optimal size in all these four cities. And comparing the real size, we can hardly find whether the optimal or the market size is more precise to the real situation. Beijing and Tokyo seem to satisfy the optimal market size situation while Shanghai and New Delhi are on the opposite.





From the above two graphics, we can find that the Tokyo has the highest density among these four cities while Beijing has the lowest, which is the result of small city area. It is noticed that after environment protection procedure is taken into account, the population density among these four cities between 15 km and 25 km becomes closer. Tokyo, for instance, which has the highest center density, experience sharp density decline when we step away from CBD after considering environment protection. Beijing, on the contrary, the decline is not as steep as Tokyo as the city size is considerably larger than Tokyo to have more space to adjust freely.

City	Beijing	Shanghai	Tokyo	New Delhi
Average density increase %	40.9	18.0	21.6	18.6

From the above table it could be concluded that big cities have the greatest density to increase thanks to more place to adjust population density. In small cities, on the other hand, workers have no such "freedom" to adjust.

5 Concluding Remarks

Our method to consider environment cleaness level into the model is to endogenize it into the utility function and budget constraint. The collected tax is used to raise environment quality which is contained in the utility function. The population density and city size are consequently affected by such government action. It is natural that the marginal utility gained from raising cleaness quality must equal marginal utility loss of disposable income. The difference between the social marginal cost of environment protection and private marginal cost makes employees choose to live at different working places, which leads to different city cize. From the discussion of the sign of λ_3, comparative static analysis result shows the endogization of anti-pollution action makes the population density at any place increase. Land owners suffer rent loss since the disposable income of rent payers decrease, which is the direct result of tax collection. Such loss could be slight if the marginal utility contribution of cleaness level is weak when the overall atmosphere is clean or the living standard is adequately low. Numerical simulation on Beijing, Shanghai, Tokyo and New Delhi shows the population density indeed increase after pollution is no longer ignored. Also, urban area which is 5 to 15 kilometers away from the central business district is the most influenced region comparing to the center. We assume the protection efficiency depends on the expense per area per capita. As a result, people living in such area suffer the greatest impact on environment protection while workers near CBD might neglect the effect thanks to the high density level. Also from the numerical result density level in big cities is easier to adjust because large city size provides more flexibility. This paper only discuss the situation of uniform tax rate among urban area. However, it is not fair for people living in a high density area to pay the same rate since the marginal utility gain from protection is low for them. It remains for future research if flexible tax rate policy is allowed in the model, which makes fiscal policy more precise.

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