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ECO 480A Econometrics

Homework 3 Solution

Due Date: 11/15/2013

8-1

a. The sample mean [$\bar{X}$, μ] is an unbiased estimate of the population mean [$\bar{X}$, μ]—assuming the sample is [random, very large].

b. $\bar{X}$ fluctuates from sample to sample with a standard deviation equal to [σ/n, $\sigma/\sqrt{n}$], which is also called the [standard error SE, population standard deviation].

c. If we make an allowance of about [$\sqrt{n}$, 2] standard errors on either side of $\bar{X}$, we obtain an interval wide enough that it has a 95% chance of covering the target μ. This is called the 95% confidence interval for [$\bar{X}$, μ].

d. A statistician who constructed a thousand of these 95% confidence intervals over his lifetime would miss the target [practically never, about 50 times, about 950 times]. Of course, he [would, would not] know just which times these were.

e. For greater confidence such as 99%, the confidence interval must be made [narrower, wider].

8-3

$$\mu = \bar{X} + 1.96 \frac{\sigma}{\sqrt{n}} = 3700 \pm 1.96 \frac{6000}{\sqrt{225}} \qquad (8\text{-}9)$$

$$= 3700 \pm 784$$

$$\text{Total} = N\mu = 2700\ (3700 \pm 784)$$

$$= 9{,}990{,}000 \pm 2{,}116{,}800 \simeq 10{,}000{,}000 \pm 2{,}000{,}000$$

We could get a slightly more accurate SE by using the correction factor for small populations:

$$SE = \frac{\sigma}{\sqrt{n}} \sqrt{\frac{N-n}{N-1}} = \frac{6000}{\sqrt{225}} \sqrt{\frac{2700-225}{2700-1}} = 383 \qquad (6\text{-}25)$$

Then we finally get:

$$\text{Total} = 2700\ (3700 \pm 1.96 \times 383)$$
$$= 9{,}990{,}000 \pm 2{,}027{,}000$$
$$\simeq 10{,}000{,}000 \pm 2{,}000{,}000 \text{ still}$$

8-5 True.

8-7 a. Our calculator gives $\bar{X} = 99.8$ and $s = 54.7$. Since df = n-1 = 4, Table V gives $t_{.025} = 2.78$. Then

$$\mu = \bar{X} \pm t_{.025} \frac{s}{\sqrt{n}}$$

$$= 99.8 \pm 2.78 \frac{54.7}{\sqrt{5}} = 99.8 \pm 67.9$$

b. $\text{Total} = N\mu = 50(99.8 \pm 67.9)$

$$= 4990 \pm 3397 \simeq 5000 \pm 3400$$

c. Since the confidence interval runs from about 1600 to 8400, it nicely covers the true area of 3620.

8-11 a.

WOMEN			MEN		
X_1	$X_1-\bar{X}_1$	$(X_1-\bar{X}_1)^2$	X_2	$X_2-\bar{X}_2$	$(X_2-\bar{X}_2)^2$
9	-2	4	16	0	0
12	+1	1	19	3	9
8	-3	9	12	-4	16
10	-1	1	11	-5	25
16	+5	25	22	6	36
$\bar{X}_1 = \frac{55}{5} = 11$	0 ✓	40	$\bar{X}_2 = \frac{80}{5} = 16$	0 ✓	86

$$s_p^2 = \frac{\Sigma(X_1 - \bar{X}_1)^2 + \Sigma(X_2 - \bar{X}_2)^2}{(n_1 - 1) + (n_2 - 1)} \qquad (8\text{-}21)$$

$$= \frac{40 + 86}{4 + 4} = \frac{126}{8} = 15.75$$

Since d.f. = 8, $t_{.025} = 2.31$. Thus

$$(\mu_1 - \mu_2) = (\bar{X}_1 - \bar{X}_2) \pm t_{.025}\, s_p \sqrt{\frac{1}{n_1} + \frac{1}{n_2}} \qquad (8\text{-}20)$$

$$= (11 - 16) \pm 2.31 \sqrt{15.75} \sqrt{\frac{1}{5} + \frac{1}{5}}$$

$$\mu_W - \mu_M = -5 \pm 5.80 \simeq -5 \pm 6$$

That is, women are estimated to earn 5 thousand dollars <u>less</u> (because of the minus sign).

We calculated the differences of women relative to men (in that order) merely because that was the given order. We could just as easily have used the other order, and found

$$\mu_M - \mu_W = (16 - 11) \pm \ldots$$

$$= +5 \pm 5.80 \simeq +5 \pm 6$$

That is, men are estimated to earn 5 thousand dollars more than women. This

8-15 a.

Treatment	Control	Difference D	$D - \bar{D}$	$(D - \bar{D})^2$
68	65	3	0	0
65	62	3	0	0
66	64	2	-1	1
66	65	1	-2	4
67	65	2	-1	1
66	64	2	-1	1
66	59	7	4	16
64	63	1	-2	4
69	65	4	1	1
63	58	5	2	4
		$\bar{D} = \frac{30}{10} = 3$	0 ✓	$s^2 = \frac{32}{9} = 3.56$

$$\Delta = \bar{D} \pm t_{.025} \frac{s_D}{\sqrt{n}} = 3 \pm 2.26 \sqrt{\frac{3.56}{10}} = 3.00 \pm 1.35$$

8-30

$\overline{X}_1$	$\overline{X}_2$	$D = \overline{X}_1 - \overline{X}_2$	$D - \overline{D}$	$(D - \overline{D})^2$
23	20	3	1	1
17	16	1	-1	1
16	14	2	0	0
20	18	2	0	0

$\overline{D} = 2 \qquad S_D^2 = 2/3$

$$D = 2 \pm 3.18 * \sqrt{0.67} / \sqrt{4} = 2 \pm 1.3 = (0.7, 3.3)$$

9-1

Problem	95% CI	Is it discernible, i.e., Does estimate stand out above "fog"?
8-24	$\Delta = -4.0 \pm 5.8$	no
8-25a	$\mu_2 - \mu_1 = 13 \pm 21$	no
8-26b	$\pi_2 - \pi_1 = -45\% \pm 9\%$	yes!

9-7.

a. I; α

b. II; β

c. α; β

9-9.

a. $H_0: \ \pi_0 = 0.10$; $H_A: \ \pi_0 > 0.10$

b. Assume that $H_0: \ \pi_0 = 0.10$ is true.

Then $\Pr(P \geq P_C) = 0.09$, that is,

$$\Pr\left(z \geq \frac{P_C - \pi_0}{\sqrt{\pi_0(1-\pi_0)/n}} \right) = 0.09 .$$

Table IV gives $\Pr(z \geq 1.34) = 0.09$.

Thus $\dfrac{P_C - \pi_0}{\sqrt{\pi_0(1-\pi_0)/n}} = 1.34$ ➔ $\dfrac{P_C - 0.10}{\sqrt{0.10(1-0.10)/100}} = 1.34$

We can get $P_C = 0.14$ (14%).

Thus we can reject H_0 if the proportion of defective exceeds the critical value 14%.

(This also means that the p-value for H_0 is smaller than 9%.)

c. From part b, the shipments with 25%, 16%, 24%, and 21% should be rejected.

9-12.

Assume that $H_0: \ (\pi_1 - \pi_2) = 0$ is true.

Here let $P_1 = 0.34$ and $P_2 = 0.24$.

Then calculate $\Pr((P_1 - P_2) \geq 0.10)$, that is,

$$\Pr\left(z \geq \frac{0.10 - (\pi_1 - \pi_2)}{SE \text{ of } (P_1 - P_2)} \right)$$

$$= \Pr\left(z \geq \frac{0.10 - 0}{\sqrt{\frac{0.34(1-0.34)}{510}} + \sqrt{\frac{0.24(1-0.24)}{420}}} \right).$$

We get:

$\Pr(z \geq 3.382)$.

Table IV gives $\Pr(z \geq 3.382) \approx \Pr(z \geq 3.4) = 0.000337$.

b. From part a, the p-value is smaller than 5%. So we can reject $H_0: \ (\pi_1 - \pi_2) = 0$.

Thus the difference is statistically significant at the 95 % confidence level.

Q1

8.13

a. $H_o: \mu \geq 30,000$
$H_A: \mu < 30,000$

b. For alpha = .05 and a one tailed , lower tail test, the critical value is z = -1.645

c. $$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{29,750 - 30,000}{\frac{2,500}{\sqrt{100}}} = \frac{-250}{250} = -1.00$$

Since z = -1.00 > -1.645, do not reject the null hypothesis.

d. A Type II error could have been made since the null hypothesis was not rejected.

Q2

8.15.

a. H_o: $\mu \leq 6$ days
H_a: $\mu > 6$ days

b. $$z = \frac{\bar{x} - \mu}{\frac{s}{\sqrt{n}}} = \frac{6.65 - 6.00}{\frac{1.5}{\sqrt{40}}} = \frac{.65}{.2372} = 2.7406$$

$z_{.025} = 1.96$

Since 2.7406 > 1.96 reject H_o and conclude that the mail-order business is not achieving its goal.