Ponder This Challenge for Oct. 2013

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Ponder This Challenge: Color the squares of an 8x8 check-board in two colors (black and white) and calculate the percentage of black squares in every one of the 8 rows, 8 columns and 30 diagonals.

Our challenge this month is to find such coloring which gives the same percentage for every row; different one for every column and as many different numbers in the diagonals as possible.

Solution. Suppose $a_i (i = 1, \cdots, 8)$ represent the number of black squares in i^{th} column. Then $\{a_i\}_{i=1}^{8} = \{0, 1, \cdots, 8\} \setminus \{k\}, k \in \{0, 1, \cdots, 8\}$. Notice each row has the same number of black square, we have $8 | \sum_{i=1}^{8} a_i$. Therefore $k = 4$. $\{a_i\}_{i=1}^{8} = \{0, 1, 2, 3, 5, 6, 7, 8\}$ and the black squares in each row are four.

When we focus on the diagonals, we find the possible ratio of black square is {0,1,1/2,1/3,2/3,1/4,3/4,1/5,2/5,3/5,4/5,1/6,5/6,1/7,2/7,3/7,4/7, 5/7,6/7,1/8,3/8,5/8,7/8}.But in 8*8 check-board we only have four diagonals with length 7 and two diagonals with length 8. So we can only reach at most four ratios in $\{1/7, 2/7, 3/7, 4/7, 5/7, 6/7\}$ and two ratios in $\{1, 8, 3/8, 5/8, 7/8\}$. The following coloring method satisfies the maximum possible ratios in diagonals.

$$\begin{pmatrix} B & B & B & B & W & W & W & W \\ B & B & B & W & B & W & W & W \\ B & B & W & B & W & B & W & W \\ B & B & B & W & B & W & W & W \\ B & B & B & B & W & W & W & W \\ B & B & B & W & B & W & W & W \\ B & W & W & B & W & B & B & W \end{pmatrix}$$