

October 2014 Ponder This Challenge

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Ponder This Challenge: What are the last 10 decimal digits of $2^{3^{4^{5^{6^{7^{8^9}}}}}}$?

My Solution:

We have the following results:

$$2^{n+4 \cdot 5^9} = 2^n \pmod{10^{10}} \quad \forall n \geq 10$$

$$3^{n+8 \cdot 5^8} = 3^n \pmod{4 \cdot 5^9} \quad \forall n \geq 0$$

$$4^{n+2 \cdot 5^7} = 4^n \pmod{8 \cdot 5^8} \quad \forall n \geq 2$$

$$5^n = 5^7 \pmod{2 \cdot 5^7} \quad \forall n \geq 7$$

Now let $n = 6^{7^{8^9}}$.

$$\text{So then } 4^{5^n} = 390624 \pmod{8 \cdot 5^8}$$

$$\text{And } 3^{4^{5^n}} = 5765981 \pmod{4 \cdot 5^9}$$

$$\text{And } 2^{3^{4^{5^n}}} = 8170340352 \pmod{10^{10}}$$