MICROECONOMIC THEORY I. ECONOMICS 665. FALL 2014.

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October 15, 2014. TIME: 9:30 AM-10:50 AM PLACE: 424 FRONCZAK.

Books, notes or calculators are not allowed in the examination room. Please answer each question. **TOTAL POINTS = 100.**

PROBLEM 1(30 Points): A firm has two factories. The first factory can produce only in period 1 with cost function $C_1(w_1, w_2, y_1) = y_1^2 \min(w_1, 2w_2)$. Factory 2 can produce only in period 2 with cost function $C_2(w_1, w_2, y_2) = (2w_1 + 4w_2)y_2$.

(a) Derive the first factory's production function and state what kinds of returns to scale the technology has? Very briefly explain how you know your answers. **(5 points)**

(b) Derive the second factory's production function and state what kind of returns to scale the technology has? Very briefly explain how you know your answers? **(5 points)**

(c) Suppose that the firm minimizes $C_1(w_1, w_2, y_1) + \frac{1}{1+r} C_2(w_1, w_2, y_2)$ where r is the interest rate, to produce a total target output amount of 1 unit. Prove that the firm will produce all of this output in period 1 if $r = 0$. **(10 points)**

(d) At what interest rate or range of interest rates will the firm produce a target total output amount of 1 unit some of it in period 1 and some of it in period 2? **(10 points).**

PROBLEM 2 (45 Points): A profit maximizing firm has a production function $f(x_1, x_2, b)$ which is strictly concave and increasing in the two inputs x_1, x_2. The prices of the two inputs are w_1, w_2. The price of the output is p. α is a parameter beyond the firm's control.

- **(a)** Suppose that $f(x_1, x_2, \alpha)$ is **strictly concave** in α. Express this using an inequality. **(10 points)**
- **(b)** Suppose that $f(x_1, x_2, \alpha)$ is increasing in α. Prove using the Envelop Theorem that the maximized profit increases in α. **(10 points)**
- **(c)** Suppose that an increase in α increases the marginal product of input 1 leaving the marginal product of input 2 unchanged. Derive the expressions $\frac{dx_1}{d\alpha}, \frac{dx_2}{d\alpha}$, where x_1, x_2 are the profit maximizing input quantities.

  Simplify as much as possible the expressions you derived and determine their signs as much as possible. **(25 points)**

PROBLEM 3 (25 Points): Consider the following non-differentiable technology:

$f(x) = x^{1/2}$ for $0 \le x \le x_0$ and $f(x) = x_0^{1/2}$ for $x > x_0$.

(a) Draw the production function. Clearly label the diagram. **(5 points)**

(b) Prove that the firm will never choose $x = 0$ to maximize its profit. **(10 points)**

(b) Derive and correctly express the indirect profit function, $\pi(p, w)$ of this technology. **(10 points)**